CLAIMS:

1. A method for reducing impulse noise in a signal processing system, comprising the steps of:

estimating parameters of an alpha-stable distribution to model impulse noise that corrupts data signals input into a transmission medium of the signal processing system;

sampling signals from the transmission medium; said sampling step storing the sampled signals in a memory; the sampled signals having a noise component and a data component; and

computing with a prediction filter an estimate of the data components of the sampled signals using the estimated parameters of the alpha-stable distribution.

2. The method according to claim 1, wherein said sampling step is performed from a digital subscriber line (DSL).

3. The method according to claim 2, further comprising the steps of:

recording samples of impulse noise signals $\{x_1, x_2, \ldots, x_n\}$ transmitted over the transmission medium of the signal processing system; the samples of impulse noise signals recorded by said recording step having no data component; and

using the recorded samples of impulse noise to perform said estimating step.

4. The method according to claim 1, wherein said sampling step is performed using an imaging device.

5. The method according to claim 4, further comprising the steps of:

recording a first, a second, and a third image with the imaging device;

computing a difference between the first and the second image to define a centro-symmetrized difference image;

estimating a characteristic exponent of an alpha-stable distribution using the centro-symmetrized difference image; and

computing a sample of impulse noise signals by applying a centralizing transformation to the first, the second, and the third images.

6. The method according to claim 1, further comprising the steps of:

generating a synthetic noise signal with a random number generator that uses the parameters of the alpha-stable distribution as input parameters to the random number generator; and

subtracting the synthetic noise signal from the sampled signal to produce a modified sampled signal; the modified sampled signal having a noise component with a symmetric distribution.

7. The method according to claim 1, further comprising the steps of:

estimating a characteristic exponent of the alpha-stable distribution that models impulse noise corrupting the sampled signal; and

optimizing model coefficients of the prediction filter using the estimated characteristic exponent of the alpha-stable distribution.

8. The method according to claim 7, wherein said optimizing step minimizes a pth-power error criterion to optimize the model coefficients of the prediction filter.

9. The method according to claim 8, wherein said optimizing step defines the pth-power error criterion using the characteristic exponent of the alpha-stable distribution.

10. The method according to claim 8, wherein said optimizing step minimizes the pth-power error criterion by performing the following computation iteratively:

$$C(k) = (X_{ext}^T W X_{ext})^{-1} X_{ext}^T W \underline{x}_t,$$

where C are model coefficients, W is a diagonal weight matrix, and $\underline{x}_t$ is an observed signal block of L samples, where:

$$C = \begin{bmatrix} \underline{a} \\ \underline{b} \\ \underline{c} \end{bmatrix}, \text{ such that } \underline{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_N \end{bmatrix}, \ \underline{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_{\frac{N(N+1)}{2}} \end{bmatrix}, \text{ and } \underline{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_{\frac{N(N+1)(N+2)}{6}} \end{bmatrix},$$

$$\underline{x}_t = \begin{bmatrix} x[t \times L] \\ \vdots \\ x[t \times L + L - 1] \end{bmatrix}, \quad where\ t = 0, 1, 2, 3, \ \ldots$$

$X_{ext} = \left[X^{(1)} X^{(2)} X^{(3)}\right]$, such that

$$X^{(1)} = \begin{bmatrix} x[t\times L] & 0 & \cdots & 0 \\ x[t\times L+1] & x[t\times L] & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x[t\times L+L-1] & x[t\times L+L-2] & \cdots & x[t\times L+L-N] \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} x^2[t\times L] & 0 & \cdots & 0 \\ x^2[t\times L+1] & x[t\times L+1]x[t\times L] & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x^2[t\times L+N] & x[t\times L+N]x[t\times L+N-1] & \cdots & x^2[t\times L] \\ \vdots & \vdots & & \vdots \\ x^2[t\times L+L-1] & x[t\times L+L-1]x[t\times L+L-2] & \cdots & x^2[t\times L+L-N] \end{bmatrix}$$

$$X^{(3)} = \begin{bmatrix} x^3[t\times L] & 0 & \cdots & 0 \\ x^3[t\times L+1] & x^2[t\times L+1]x[t\times L] & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x^3[t\times L+N] & x^2[t\times L+N]x[t\times L+N-1] & \cdots & x^3[t\times L] \\ \vdots & \vdots & & \vdots \\ x^3[t\times L+L-1] & x^2[t\times L+L-1]x[t\times L+L-2] & \cdots & x^3[t\times L+L-N] \end{bmatrix}$$

11. The method according to claim 8, wherein said step of estimating the characteristic exponent comprises the steps of:

computing a moment of the alpha-stable distribution; and

estimating the characteristic exponent using the computed moment.

12. The method according to claim 11, wherein said step of computing the moment of the alpha-stable distribution comprises the step of computing an absolute fractional lower order moment $\hat{A}_p$ according to the following equation:

$\hat{A}_p = \frac{1}{n}\sum_{k=1}^{n} |X_k|^p$, where p is the order of the moment and X_k is one sample from a sequence of noise samples $\{X_1, X_2, \ldots, X_n\}$.

13. The method according to claim 12, wherein said step of estimating the characteristic exponent comprises the step of estimating the characteristic exponent by computing $\hat{Z} = \log \hat{A}_p^X - \log \hat{A}_p^Y$ using the computed absolute fractional lower order moment $\hat{S}_p$ by performing the steps of:

partitioning the sequence of noise samples into two parts U and V, with each part containing data samples $U_1, U_2, U_3, \ldots$ and $V_1, V_2, V_3, \ldots$, respectively;

computing the moment $\hat{A}_p^X$ by summing noise samples in the sequence of noise samples as:

$X_1=U_1+V_1, X_2=U_2+V_2, X_3=U_3+V_3, \ldots$; and

computing the moment $\hat{A}_p^Y$ by concatenating noise samples in the sequence of noise samples as:

$Y_1=U_1, Y_2=V_1, Y_3=U_2, Y_4=V_2, Y_5=U_3, Y_6=V_3, \ldots$.

14. The method according to claim 13, wherein said estimating step estimates the characteristic exponent to be equal to $\frac{p\log 2}{\hat{Z}}$ when $\hat{Z} < \frac{p\log 2}{\alpha_{\min}}$ and $\hat{Z} > \frac{p\log 2}{2}$, where p is the order of the moment.

15. The method according to claim 11, wherein said step of computing the moment of the alpha-stable distribution comprises the step of computing an absolute logarithmic moment using the following equation:

$\hat{L}_1 = \frac{1}{n}\sum_{k=1}^{n} \log|X_k|$, where X_k is one sample from a sequence of noise samples $\{X_1, X_2, \ldots, X_n\}$.

16. The method according to claim 15, wherein said estimating step estimates the characteristic exponent by computing $\hat{Z} = \hat{L}_1^X - \hat{L}_1^Y$ using the computed absolute logarithmic moment $\hat{L}_1$, by performing the steps of:

partitioning the sequence of noise samples into two parts <u>U</u> and <u>V</u>, with each part containing data samples $U_1, U_2, U_3, \ldots$ and $V_1, V_2, V_3, \ldots$, respectively;

computing the moment $\hat{L}_1^X$ by summing noise samples in the sequence of noise samples as:

$X_1=U_1+V_1$, $X_2=U_2+V_2$, $X_3=U_3+V_3$, …; and

computing the moment $\hat{L}_2^X$ by concatenating noise samples in the sequence of noise samples as:

$Y_1=U_1$, $Y_2=V_1$, $Y_3=U_2$, $Y_4=V_2$, $Y_5=U_3$, $Y_6=V_3$, … .

17. The method according to claim 16, wherein said estimating step estimates the characteristic exponent to be equal to $\frac{\log 2}{\hat{Z}}$ when $\hat{Z} < \frac{\log 2}{\alpha_{\min}}$ and $\hat{Z} > \frac{\log 2}{2}$.

18. The method according to claim 16, wherein said computing step computes a signed logarithmic moment using the following equation:

$$\hat{\Lambda} = \frac{1}{n}\sum_{k=1}^{n} \mathrm{sign}(X_k)\log|X_k|.$$

19. The method according to claim 18, further comprising the step of estimating a symmetry parameter of an alpha-stable distribution by computing

$$\frac{\tan((\hat{\Lambda}/\hat{L}_1)\,\alpha\pi/2))}{\tan(\alpha\pi/2)}.$$

20. The method according to claim 1, wherein said computing step extrapolates with the prediction filter an estimate of the data components of the sampled signals using the estimated parameters of the alpha-stable distribution.

21. The method according to claim 1, wherein said computing step interpolates with the prediction filter an estimate of the data components of the sampled signals using the estimated parameters of the alpha-stable distribution.

22. The method according to claim 1, further comprising the step of computing a moment of the alpha-stable distribution.

23. The method according to claim 22, wherein said step of computing a moment comprises the step of computing one of:

an absolute fractional lower order moment $\hat{A}_p = \frac{1}{n}\sum_{k=1}^{n} |X_k|^p$,

a signed fractional lower order moment $\hat{S}_p = \frac{1}{n}\sum_{k=1}^{n} \mathrm{sign}(X_k)|X_k|^p$,

a signed logarithmic moment $\hat{\Lambda} = \frac{1}{n}\sum_{k=1}^{n} \text{sign}(X_k)\log|X_k|$,

a first absolute logarithmic moment $\hat{L}_1 = \frac{1}{n}\sum_{k=1}^{n} \log|X_k|$,

a second absolute logarithmic moment $\hat{L}_2 = \frac{1}{n-1}(\sum_{k=1}^{n} \log|X_k| - \hat{L}_1)^2$,

a third absolute logarithmic moment $\hat{L}_3 = \frac{1}{n-1}(\sum_{k=1}^{n} \log|X_k| - \hat{L}_1)^3$,

an empirical characteristic function moment $\hat{\varphi}(p) = \frac{1}{n}\sum_{k=1}^{n} e^{jpX_k}$,

a first upper extreme value moment $\hat{\overline{Y}}_1 = \frac{1}{n/r}\sum_{k=1}^{n/r} \overline{K}_k$,

a first lower extreme value moment $\underline{\hat{Y}_1} = \frac{1}{n/r}\sum_{k=1}^{n/r} \underline{K}_k$,

a second upper extreme value moment $\hat{\overline{Y}}_2 = \frac{1}{(n/r)-1}\sum_{k=1}^{n/r} (\overline{K}_k - \hat{\overline{Y}}_1)^2$, and

a second lower extreme value moment $\underline{\hat{Y}_2} = \frac{1}{(n/r)-1}\sum_{k=1}^{n/r} (\underline{K}_k - \underline{\hat{Y}_1})^2$,

where:

X_k is one sample from a sequence of noise samples $\{X_1, X_2, \ldots, X_n\}$,

p is the order of the moment,

$\overline{K}_k = \max\{\log X_{r(k-1)+1}, \log X_{r(k-1)+2}, \cdots \log X_{r(k-1)+r-1}\}$,

$\underline{K}_k = \max\{\log\ -X_{r(k-1)+1}, \log\ -X_{r(k-1)+2}, \cdots \log\ -X_{r(k-1)+r-1}\}$, and

r is a block length of data.

24. The method according to claim 23, further comprising the step of computing a characteristic exponent α of the alpha-stable distribution.

25. The method according to claim 24, wherein said step of computing the characteristic exponent α of the alpha-stable distribution comprises the step of computing one of:

a first auxiliary variable $\hat{Z} = \log \hat{A}_p^X - \log \hat{A}_p^Y$,

a second auxiliary variable $\hat{Z} = \hat{L}_1^X - \hat{L}_1^Y$, and

a third auxiliary variable $\hat{Z} = \left(1 - \frac{\hat{L}_3}{1.2020569}\right)^{-1/3}$, and

a fourth auxiliary variable $\hat{Z} = \frac{\pi}{2\sqrt{6}}\left(\frac{1}{\overline{Y}_2} + \frac{1}{\underline{Y}_2}\right)^{1/2}$.

26. The method according to claim 24, further comprising the step of computing a symmetry parameter β of the alpha-stable distribution.

27. The method according to claim 26, wherein said step of computing the symmetry parameter β of the alpha-stable distribution comprises the step of computing one of:

a first estimate of the symmetry parameter $\hat{\beta} = \frac{\tan((\hat{\Lambda} / \hat{L}_1)\, \alpha\pi/2))}{\tan(\alpha\pi/2)}$, and

a second estimate of the symmetry parameter $\hat{\beta} = 1 - \dfrac{2}{\exp(\alpha(\bar{Y}_1 - \underline{Y}_1))}$.

28. The method according to claim 24, further comprising the step of computing a dispersion γ of the alpha-stable distribution.

29. The method according to claim 28, wherein said step of computing the dispersion γ of the alpha-stable distribution comprises the step of computing

$\hat{\gamma} = \left(\dfrac{\Gamma(1-p)\cos(p\pi/2)}{\Gamma(1-p/\alpha)\cos(p\theta/\alpha)} \right)^{\alpha/p} |\cos\theta|$, where $\theta = \arctan(\beta \tan(\alpha\pi/2))$.

30. The method according to claim 24, further comprising the step of computing a location parameter δ of the alpha-stable distribution.

31. The method according to claim 30, wherein said step of computing the location parameter δ of the alpha-stable distribution comprises the step of computing a fractile f of the sequence of noise, where $f = \dfrac{1}{2} - \dfrac{\theta}{\pi\alpha}$, $\theta = \arctan(\beta \tan(\alpha\pi/2))$.

32. The method according to claim 1, further comprising the steps of:

computing a characteristic exponent α of the alpha-stable distribution

computing a location parameter δ of the alpha-stable distribution,

computing a symmetry parameter β of the alpha-stable distribution, and

computing a dispersion γ of the alpha-stable distribution.

33. The method according to claim 32, wherein said step of computing the characteristic exponent α and said step of computing the dispersion γ of the alpha-stable distribution comprises the step of computing the following matrix equation

$$\begin{bmatrix} \sum_{j=1,k=1}^{j=m,k=m} \log(t_j)\log(t_k)C_{jk} & \sum_{j=1,k=1}^{j=m,k=m} \log(t_k)C_{jk} \\ \sum_{j=1,k=1}^{j=m,k=m} \log(t_k)C_{jk} & \sum_{j=1,k=1}^{j=m,k=m} C_{jk} \end{bmatrix} \begin{bmatrix} \alpha \\ \log\gamma \end{bmatrix} = \begin{bmatrix} \sum_{j=1,k=1}^{j=m,k=m} \log(t_k)\psi_j C_{jk} \\ \sum_{j=1,k=1}^{j=m,k=m} \psi_j C_{jk} \end{bmatrix},$$

where

$\{t_1, t_2, \cdots, t_m\}$ is some set of positive real numbers,

C equals W^{-1} where W is an *mxm* matrix with elements

$$W_{jk} = \{\exp(\gamma'[t_j^{\alpha'} + t_k^{\alpha'} - t_j^{\alpha'} t_k^{\alpha'}]) - 1\} / \{\gamma'^2 t_j^{\alpha'} t_k^{\alpha'}\}$$

and α', γ' are existing estimates of the characteristic exponent and dispersion, and

where $\psi_k = \log(-\log(\mathrm{Re}\,\hat{\varphi}(t_k)))$ for sequence Y for $k = 1, \cdots, m$, and

Y is a centro-symmetrized version of the sequence of noise samples.

34. The method according to claim 33, wherein said step of computing the location parameter δ and said step of computing the symmetry parameter β of the alpha-stable distribution comprises the step of computing the following matrix equation:

$$\begin{bmatrix} \kappa^2 \sum_{j=1,k=1}^{j=m,k=m} (t_j t_k)^{\alpha-1} C_{jk} & \kappa \sum_{j=1,k=1}^{j=m,k=m} t_k^{\alpha-1} C_{jk} \\ \kappa \sum_{j=1,k=1}^{j=m,k=m} t_k^{\alpha-1} C_{jk} & \sum_{j=1,k=1}^{j=m,k=m} C_{jk} \end{bmatrix} \begin{bmatrix} \beta \\ \delta \end{bmatrix} = \begin{bmatrix} \sum_{j=1,k=1}^{j=m,k=m} t_k^{\alpha-1} \omega_j C_{jk} \\ \sum_{j=1,k=1}^{j=m,k=m} \omega_j C_{jk} \end{bmatrix},$$

where $\kappa = \gamma \tan\left(\frac{\alpha\pi}{2}\right)$,

C equals W^{-1} where W is the *mxm* matrix with elements

$$W_{sr} = \frac{1}{2(R_s^2 + I_s^2)(R_r^2 + I_r^2)t_s t_r}\{R_{s+r}(I_s I_r - R_s R_r) + R_{s-r}(I_s I_r + R_s R_r) + I_{s+r}(I_s R_r + R_s I_r) + I_{s-r}(I_s R_r - R_s I_r) - 4R_s I_s R_r I_r\}$$

where

$$R_s = \mathrm{Re}(\varphi(t_s)), I_s = \mathrm{Im}(\varphi(t_s)), R_{s+r} = \mathrm{Re}(\varphi(t_s + t_r)),$$

where $\varphi(t) = \exp(-\gamma |t|^\alpha (1 + j\beta' \tan\frac{\alpha\pi}{2}) + jt\delta')$ for existing estimates β', δ' of the skew and location parameters, and

$\omega_k = -\mathrm{Im}\log(\hat{\varphi}(t_k)) / t_k$ for the sequence of noise samples.

35. An apparatus for reducing impulse noise in a signal processing system, comprising:

a parameter estimation module for estimating parameters of an alpha-stable distribution; the alpha-stable distribution modeling impulse noise that corrupts data signals input into a transmission medium of the signal processing system;

a memory for accumulating sampled signals output from the transmission medium; the sampled signals having a noise component and a data component; and

a signal estimation module for computing an estimate of the data component of the sampled signals output from the transmission medium using the estimated parameters of the alpha-stable distribution.

36. The apparatus according to claim 35, wherein said signal estimation module further comprises:

a prediction filter for estimating, using model coefficients, the data components of the sampled signals output from the transmission medium; the estimated data components corresponding to an estimation of the data signals input into the transmission medium; and

a coefficient optimization module for optimizing the model coefficients of the prediction filter using one of the estimated parameters of the alpha-stable distribution received from said parameter estimation module.

37. The apparatus according to claim 36, wherein the dependence of the prediction filter output on the model coefficients is linear.

38. The apparatus according to claim 37, wherein said prediction filter is a Volterra filter.

39. The apparatus according to claim 36, wherein the dependence of the prediction filter output on the data components of the sampled signals input into said prediction filter is non-linear.

40. The apparatus according to claim 36, wherein said parameter estimation module further comprises:

means for computing moments of the alpha-stable distribution; and

means for estimating the parameters of the alpha-stable distribution with the computed moments.

41. The apparatus according to claim 40, wherein said parameters estimation module further comprises means for transforming the alpha-stable distribution to obtain deskewed alpha-stable random variables.

42. The apparatus according to claim 40, wherein said parameters estimation module further comprises means for transforming the alpha-stable distribution to obtain centralized alpha-stable random variables.

43. The apparatus according to claim 36, wherein said parameter estimation module adaptively estimates the parameters of the alpha-stable distribution.

44. The apparatus according to claim 36, wherein said coefficient optimization module uses the parameters of the alpha-stable distribution to specify a minimum dispersion error criterion for determining the model coefficients of said prediction filter.

45. The apparatus according to claim 44, wherein said coefficient optimization module minimizes a cost function defined by the minimum dispersion error criterion to optimize the model coefficients of said prediction filter.

46. The apparatus according to claim 45, wherein said coefficient optimization module minimizes a p^{th}-power error criterion to optimize the model coefficients of said prediction filter.

47. The apparatus according to claim 36, wherein transmission medium into which the data signals are input is a twisted pair.

48. The apparatus according to claim 35, wherein the signal processing system operates in a digital subscriber line (DSL).

49. The apparatus according to claim 35, wherein the signal processing system operates in an imaging device.